PhD position
Quantum turbulence simulations using a vortex filament model

Summary Superfluids, such as liquid helium at very low temperatures, are characterised by an absence of viscosity at the absolute zero. In this limit, they are effectively composed of very thin vortex filaments about which the velocity circulation takes discrete values. The complex interaction between these quantum vortices can lead to a state known as quantum turbulence, characterised by an extremely wide range of scales going from the vortex thickness (~1 Å) to the size of existent experimental facilities (up to ~1 metre). This state has been shown to present quantitative similarities with classical turbulent flows, and thus provides an interesting avenue for a better understanding of classical turbulence. With this in mind, simulations of the vortex filament model (VFM) will be performed in this thesis to study quantum turbulent flows. In the VFM, vortices are described as three-dimensional curves which interact according to Biot–Savart's law. In particular, a reformulation of the VFM will be used for the first time to speed-up the simulation of strongly turbulent regimes in fully periodic domains. The proposed method will be first validated by comparing simple cases (e.g. the collision between two vortices leading to their reconnection) against results known from a compressible atomic-scale model. Then, a statistical analysis of the quantum turbulent state will be performed. The main objective is to compare high-order statistics associated to extreme events to results known from classical turbulence, as an attempt to further advance the analogy between both systems. In particular, statistics associated to Eulerian and Lagrangian velocities will be considered. The motion of vortex filaments will be tracked in time to characterise the emergence and decay of extreme events. This study will serve as a first step towards an accurate description of superfluid helium flows, which will require coupling the VFM to the Navier–Stokes equations.

Keywords Fluid dynamics; turbulent flows; superfluid helium; vortex dynamics; numerical simulations

Scientific context In certain technological applications, liquid helium is used at temperatures below 2.17 K as a very efficient cooling agent. At these very low temperatures, helium exists in its superfluid state, characterised in particular by an effective viscosity which vanishes at the absolute zero. In this last limit, flows of superfluid helium are mainly described by the presence of vortices of atomic thickness, known as **quantum vortices**, about which the velocity circulation takes discrete values. In the presence of a large number of such vortices, the flow can become chaotic and turbulent (fig. 1), with a behaviour which presents quantitative similarities with classical turbulent flows.

From the point of view of applications, the most interesting regime takes place between about 1 and 2.17 K, where the quantum vortices coexist with a viscous (or *normal*) fluid, and where

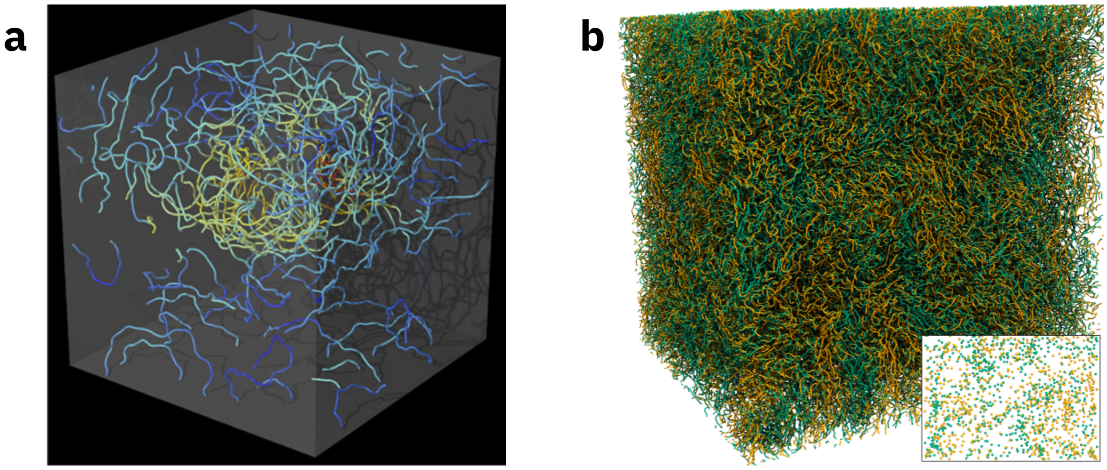

Figure 1: Quantum turbulence simulations. a Quantum vortices obtained from VFM simulations [2], coloured by their associated coarse-grained vorticity. **b** Quantum vortices obtained from simulations of the Gross–Pitaevskii model [3], valid down to the atomic scales. Colours represent the 'sign' of each vortex relative to the vertical axis.

the thermal conductivity of superfluid helium is maximal. Due to the complex coupling between normal fluid and quantum vortices and to the extremely large extent of active length scales, this regime currently escapes a full theoretical description. Recently, a promising model has been proposed for describing the macroscopic scales in this regime [1], which couples the incompressible Navier–Stokes (NS) equations for the normal fluid to a **vortex filament model** (VFM). The latter describes quantum vortices as curves in three-dimensional space [2] (fig. 1a). The velocity induced on any given vortex element follows Biot–Savart's law, and therefore requires the evaluation of an integral over all vortex filaments of the system. Hence, even in the absence of a normal fluid, the standard VFM becomes numerically very costly when a large number of vortices is present. This limitation has hindered up to now the study of highly turbulent regimes comparable to those observed in experiments.

Objectives The aim of this thesis is to **numerically study quantum turbulent flows**. For this, a novel reformulation of the VFM will be used which aims at accelerating the simulation of highly turbulent settings. The proposed approach is directly based on a family of methods commonly used in molecular dynamics simulations to compute electrostatic interactions [4]. To our knowledge, such methods have never been applied to vortex simulations, even though a strong analogy exists with the equations describing Coulomb interactions.

This thesis will focus on the **zero temperature regime**, i.e. in the absence of a normal fluid, as a first step towards an accurate description of turbulent superfluid helium flows. In this regime, the VFM can already provide important answers to fundamental open questions, concerning in particular the possible similarities and differences between classical and quantum turbulence at the absolute zero. Indeed, a quantitative link seems to exist between both systems, as strongly reaffirmed by recent works by the project supervisor and collaborators [3, 5] using an atomic-scale model (fig. 1b). Quantum turbulence is sometimes referred to as the 'skeleton' of turbulence, since its conceptually simple description in terms of vortex filaments has the potential of improving our understanding of classical turbulent flows.

The main objective is therefore to provide a **statistical characterisation of quantum turbulence**, with a special focus on extreme events which break the self-similarity of classical and quantum turbulent flows. The statistical analysis will be done both in the Eulerian and Lagrangian frames, and comparisons will be performed against known results in classical turbulence. Moreover, the tracking of vortex positions (readily available within the VFM) will enable a precise description of the mechanisms associated to extreme events, which are expected to have a strong impact on the statistics.

Profile and skills required The candidate must have a Masters degree in fluid mechanics, physics, mechanical engineering or applied mathematics. A strong background on fluid mechanics is required. Knowledge of turbulent flows and vortex dynamics will be highly welcome. A programming experience is also required (for instance in Julia, Python, Matlab, C, C++, Fortran, …). The candidate should have good oral and written communication skills in English. Skills in quantum mechanics are not required.

Supervision The PhD work will be co-supervised by Juan Ignacio Polanco (CNRS researcher) and Guillaume Balarac (professor at Grenoble INP - Ense³). The work will be carried out at the LEGI laboratory, in the MOST team, in Grenoble, France. Collaborations are planned with G. Krstulovic (Observatoire de la Côte d'Azur, Nice) and M. Gibert (Institut Néel, Grenoble).

Funding Funding for a doctoral contract has been requested to the I-MEP² doctoral school. Funding will be confirmed early June 2023. The net salary is about 1640 € during 3 years.

Start date 1 October 2023

Applying Interested candidates should contact Juan Ignacio Polanco (`juan-ignacio.polanco@univ-grenoble-alpes.fr`) before 12 May 2023 for further information.

References

[1] L. Galantucci, A. W. Baggaley, C. F. Barenghi and G. Krstulovic, 'A new self-consistent approach of quantum turbulence in superfluid helium', Eur. Phys. J. Plus **135**, 547 (2020).

[2] R. Hänninen and A. W. Baggaley, 'Vortex filament method as a tool for computational visualization of quantum turbulence', Proc. Natl. Acad. Sci. USA **111**, 4667–4674 (2014).

[3] J. I. Polanco, N. P. Müller and G. Krstulovic, 'Vortex clustering, polarisation and circulation intermittency in classical and quantum turbulence', Nat. Commun. **12**, 7090 (2021).

[4] A. Arnold and C. Holm, 'Efficient Methods to Compute Long-Range Interactions for Soft Matter Systems', in *Advanced Computer Simulation Approaches for Soft Matter Sciences II*, edited by C. Holm and K. Kremer, Advances in Polymer Science (Springer, Berlin, Heidelberg, 2005), pp. 59–109.

[5] N. P. Müller, J. I. Polanco and G. Krstulovic, 'Intermittency of Velocity Circulation in Quantum Turbulence', Phys. Rev. X **11**, 011053 (2021).